PARLIAMENT OF INDIA
(JOINT RECRUITMENT CELL)

MAIN EXAMINATION FOR POSTS OF EXECUTIVE/LEGISLATIVE/COMMITTEE/PROTOCOL OFFICER AND RESEARCH/REFERENCE OFFICER IN LOK SABHA SECRETARIAT

28th AUGUST, 2008

MATHEMATICS - Paper-I

INSTRUCTIONS : Answers must be written in English only. Candidates should attempt at least 2 questions from each section and total 5 questions. The number of marks carried by each question is indicated at the end of the question.

Time: 3 hours

Marks: 300

SECTION - A

Q1- (a) Evaluate $\lim_{x \to 0} \dfrac{(1+x)^{\frac{1}{x}} - e + \frac{1}{2}ex}{x^2}$ (15 marks)

(b) Find all asymptotes of the curve:
$$x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$$ (15 marks)

(c) Find the value of $\frac{1}{a^2}\frac{\partial^2 z}{\partial x^2} + \frac{1}{b^2}\frac{\partial^2 z}{\partial y^2}$, when $a^2x^2 + b^2y^2 - c^2z^2 = 0$. (15 marks)

(d) Show that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{1/e}$. (15 marks)

Q2- (a) Using Lagrange's Mean Value Theorem, show that
$$\frac{x}{1+x} < \log(1+x) < x, \quad \text{for } x > 0.$$ (15 marks)

(b) Find the centre of gravity of an arc of a circle of radius a subtending an angle 2α at its centre. (15 marks)

c) Prove that:

$$\int_0^{\pi/2} \sin^p x \, dx \times \int_0^{\pi/2} \sin^{p+1} x \, dx = \frac{\pi}{2(p+1)}$$ (15 marks)

d) Test the improper integral $\int_0^{\pi} \frac{\sqrt{x}}{\sin x} \, dx$ for convergence. (15 marks)

3- (a) The plane $x+y+z=1$ cuts the cylinder $x^2+y^2=1$ in an ellipse. Find the points on the ellipse that lie closest to and farthest from the origin. (15 marks)

(b) Find the volume of the region enclosed by the surfaces $z = x^2+3y^2$ and $z = 8-x^2-y^2$ (15 marks)

(c) Sketch the region of integration, determine the order of integration and evaluate the following double integral:

$$\iint_R (y-2x^2)\, dA$$, where R is the region inside the square $|x|+|y|=1$

(15 marks)

(d) Find the value of K for which the equation

$x^2 - kxy + 2y^2 + 3x - 5y + 2 = 0$ represents a pair of straight lines. Also find the lines. (15 marks)

Q4. (a) Find the equation of the plane containing the line $\frac{1}{2}(x+2) = \frac{1}{3}(y+3) = -\frac{1}{2}(z-4)$ and the point $(0,6,0)$. (15 marks)

(b) A sphere S has points $(0,1,0)$, $(3,-5,2)$ at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphe S with the plane $5x - 2y + 4z + 7 = 0$ as a great circle (15 marks)

(c) Show that the cone $yz + zx + xy = 0$ cuts the sphere, $x^2 + y^2 + z^2 = a^2$ in two equal circles and find their area. (15 marks)

(d) Express the matrix

$$A = \begin{bmatrix} 1+2i & 2 & 5-5i \\ 2i & 2+i & 4+2i \\ -1+i & -4 & 7 \end{bmatrix}$$

as sum of a Hermitian and a skew-Hermitian matrix. (15 marks)

15- Attempt <u>any five</u> parts :

(a) Show that the vectors $(1,2,3,4)$, $(0,1,-1,2)$, $(1,5,1,8)$, $(3,7,8,14)$ in $\mathbb{R}^4$ are linearly dependent over $\mathbb{R}$.

(b) What is the dimension of $\mathbb{R}$ (all reals) over $\mathbb{Q}$ (all rationals)? Justify your answer.

(c) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(1,1) = (1,3)$, $T(-1,1) = (3,1)$. Find $T(a,b)$ for any $(a,b) \in \mathbb{R}^2$.

(d) Let U be a linear transformation on a vector space V satisfying $U^2 - U + I = 0$. Show that U is invertible.

(e) Show that every skew-symmetric matrix of odd order is singular.

(f) Find the characteristic equation of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 0 & -1 \end{pmatrix}$ and hence compute its cube.

$(12 \times 5 = 60 \text{ Marks})$

SECTION-B

6 - (a) Find the differential equation of all tangent lines to the parabola $y = x^2$. What is its order and its degree? (15 marks)

(b) Solve: $a(x\,dy - y\,dx) = x\,dx + y\,dy$. (15 marks)

(c) Use the method of variation of parameters to solve : $\dfrac{d^2y}{dx^2} + 4y = \sec^2 2x$. (15 marks)

(d) Solve : $e^{4x}(p-1) + e^{2y}(p^2) = 0$, where $p = \dfrac{dy}{dx}$ (15 marks)

7 - (a) If D denotes $\dfrac{d}{dx}$, solve the following:

$$(x^2D^2 - xD + 2)\,y = x \log x$$ (15 marks).

(b) State Serret - Frenet's formulae. Calculate the curvature and torsion of the curve $\vec{r} = \vec{r}(u)$ in terms of u, where $\vec{r}(u) = (u, u^2, u^3)$ (15 marks)

(c) Find the unit outward drawn normal to the surface $(x-1)^2 + y^2 + (z+2)^2 = 9$ at the point $(3, 1, -4)$ (15 marks)

7) If $p = \frac{dy}{dx}$, solve the following:

(15 marks)

$$p^3 + p = e^y.$$

8 - (a) State Gauss Divergence Theorem. Use it to evaluate $\iint\limits_S \vec{F}\cdot\hat{n}\, dS$, where $\vec{F} = 2xy\,\hat{i} + yz^2\,\hat{j} + xz\,\hat{k}$ and S is the surface of the parallelopiped bounded by $x=0,\ y=0,\ z=0,\ x=2,\ y=1$ and $z=3$.

(15 marks)

(b) Verify Green's Theorem in the plane for $\int\limits_C (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$, where C is the boundary of the region bounded by $x=0,\ y=0$ and $x+y=1$.

(15 marks)

(c) If $\vec{A}$ is a differentiable vector function and ϕ is a differentiable scalar function, show that

(15 marks)

$$\text{curl}\,(\phi\vec{A}) = \text{grad}\,\phi \times \vec{A} + \phi\,\text{curl}\,\vec{A}.$$

(d) Let $\vec{r}(t) = (\sin t)\,\hat{i} + t\,\hat{j} + (\cos t)\,\hat{k}$, $t \geq 0$; denote the position vector of a particle in space at time t. Find the time (or times) in the given time interval when the velocity and acceleration vectors are orthogonal. (15 marks)

Q9 – (a) Two equal uniform rods AB and AC, each of length $2b$, are freely jointed at A, and rest on a smooth vertical circle of radius a. Show that if 2θ is the vertical angle between them, then $b\sin^3\theta = a\cos\theta$. (15 marks)

(b) If a conservative system is in equilibrium, then show that in any infinitesimal displacement satisfying the constraints, the change in potential energy is zero. (15 marks)

(c) A uniform ladder rests in limiting equilibrium, with its lower end on a rough horizontal plane and its upper end against a smooth wall. If θ is the inclination of the ladder to the vertical, prove that

$$\theta = \tan^{-1}(2\mu)$$

where μ denotes the coefficient of friction (15 marks)

(d) ABC is a triangular lamina with side AB in the surface of a heavy homogeneous liquid. A point D is taken on AC such that the thrusts on the areas ABD and DBC are equal. Prove that $AD : AC = 1 : \sqrt{2}$ (15 marks)

Q10- (a) An ellipse is just immersed in water with its major axis vertical. Show that if the centre of pressure coincides with the focus, then the eccentricity of the ellipse must be $\frac{1}{4}$. (15 marks)

(b) A particle is in equillibrium under the action of six forces. Three of these forces are reversed, and the particle remains in equillibrium. Prove that it will still remain in equillibrium if these three forces are removed altogether. (15 marks)

(c) Let $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$. Show that $\vec{F}$ is irrotational and that $\vec{F} = \text{grad}\,\phi$, for some scalar function ϕ. Find ϕ. (15 marks)

(d) Find the directional derivative of $\phi = xy + yz + zx$ at $(1,2,0)$ in the direction of $\hat{i} + 2\hat{j} + 2\hat{k}$. (15 marks)

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28^{th} AUGUST, 2008

MATHEMATICS - Paper-II

INSTRUCTIONS : This question paper consists of 10 questions in two sections A and B. Attempt at least two questions from each section and a total of five questions. All questions carry equal marks. Questions must be answered in English only.

Time: 3 hours **Marks: 300**

SECTION – A

1. An upper triangular matrix is said to be uni upper triangular if all the entries on its main diagonal are 1. Let G be the set of all 3 x 3 uni upper triangular matrices with entries from the field Z_2 of integers modulo 2.

   (a) Prove that G is a non Abelian group.
   (b) Determine the order of G.
   (c) Find the centre of G.

2. (a) Define an integral domain. Define a field. Give an example of an integral domain which is not a field. Prove that a finite integral domain is a field.

   (b) Find integers x and y to satisfy

   $$42823x + 6409y = 17$$

   Also explain the method.

   (c) Construct a finite field having 8 elements.

3. (a) Determine all the critical points for extrema of the function $f(x,y) = \sin x + \sin y + \sin(x+y)$ on and inside the square region $S = \{(x,y) \mid 0 \le x \le \pi/2,\ 0 \le y \le \pi/2\}$.

   Also determine the maximum and minimum value of the function on S.

   (b) Find the area of that part of the surface of the cylinder $x^2 + y^2 = ax$ which lies above the (x, y) –plane and is bounded above by the paraboloid $x^2+y^2+z = a^2$ (a>0).

4. (a) If f(z) is analytic in a domain D and $|f(z)|$ is a constant in D, than prove that f(z) is a constant function in the domain D.

   (b) Evaluate the integral in the counter clockwise direction along C:

   $$\oint_C dz/(z^2+4)^2,$$

   Where C is given by $x^{2/3} + y^{2/3} = a^{2/3}$ (a> 0).

   (c) Find the bilinear map w = f(z) that maps the points $Z_1 = -2$, $Z_2 = -1-i$ and $Z_3 = 0$ onto $w_1 = -1$, $w_2 = 0$ and $w_3 = 1$, respectively.

5. (a) A maximize linear programming problem in $\mathbb{R}^2_+$ with constraints of the form $Ax \le b$ has its optimal solution given by the following incomplete table:

C_B	B	X_B	Y_1	Y_2	Y_3	Y_4
-	a_2	1	-	-	-	-
-	a_1	-	-	-	-	-
-	-	z=7	-	-	3/8	7/8

It is given that $[a_1, a_2]^{-1} = \begin{pmatrix} 3/8 & -1/8 \\ -1/8 & 3/8 \end{pmatrix}$,

(63)

Where a_1, a_2 are the first and the second columns of matrix A. Complete the simplex table.

(b) Write the dual of the following primal linear programming problem (P)

$$\left.\begin{array}{l} \max z = c^t x \\ \text{subject to } Ax \leq b \\ \qquad x \geq 0 \end{array}\right\} \quad (P)$$

Show that the dual problem possesses an optimal solution if any one of the following is satisfied:

(i) $b \geq 0$ and the dual problem has a feasible solution
(ii) $b > 0$ and $A = (a_{ij})$, $a_{ij} > 0 \ \forall i, j$.

SECTION – B

6 (a) Convert the Laplace's equation in two dimensions in cartesian to the polar coordinates.

(b) Form a partial differential equation by eliminating a, b, c from

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

(c) Using Charpit's method find the complete integral of the equation :

$$(p^2 + q^2)\, y = qz$$

7 (a) Find the unique quadratic polynomial $p(x)$ such that $p(1) = 1$, $p(3) = 27$, $p(4) = 64$, using
(i) Lagrange's interpolation formula
(ii) Newton's divided difference formula
(iii) Aitken's iterated interpolation formula.

Determine p (3)

(b) Solve the following system of equations:
(i) by the Gauss elimination method
(ii) by decomposing the coefficient matrix A= Lu

$$4x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 - 2x_3 = 4$$
$$3x_1 + 2x_2 - 4x_3 = 6$$

8 (a) The following table gives values of $\sqrt{x}$ corresponding to certain values of x:

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
$\sqrt{x}$	1.00000	1.02470	1.04881	1.07238	1.09544	1.11803	1.14017

Apply Simpson's $^1/_3$ – rule to evaluate $\int_{1.00}^{1.30} \sqrt{x}\, dx$.

(b) Discuss the Newton Raphson method for solving $f(x) = 0$, where $f'(x)$ is a continuous function. Apply the method to find a real root of the equation: $x^3 + x - 1 = 0$, Correct up to 3 decimal places.

9 (a) (i) Multiply the binary numbers 101101 and 11001.

(ii) Divide the binary number 10011001 by the binary number 1011. Write the corresponding quotient and remainder in the binary form

(iii) Divide the hexadecimal number $(1\ 3\ 1\ B\ 6\ C\ 3)_{16}$ by $(1\ A\ 2\ F)_{16}$.

(b) Represent the binary number (0.000010101) in the normalised floating point representation, given that (i) word size is 16 bits (ii) Mantissa is stored in 9 bits (iii) exponent is stored in 7 bits, (iv) both mantissa and exponent are stored in signed magnitude form.

What is the largest and the smallest number in magnitude that can be represented in the above format?

(c) Give the truth table of XOR Gate for 3 inputs.

(d) Represent $(-23)_{10}$ in binary form as:

(i) signed magnitude representation

(ii) One's compliment form

(iii) Two's compliment form.

It is given that the word size is 8 bits.

10. (a) The motion of a fluid flow be given by the velocity vector as:

$$\vec{q} = \frac{a^2(xj - yi)}{x^2+y^2}$$, where a is a constant.

(i) Test the motion if it is a possible motion of an incompressible fluid. If so, determine the equation of streamlines.

(ii) Test whether the motion is of potential kind, and if so, determine the velocity potential.

(b) Liquid flows through a pipe whose surface is the surface of revolution of the curve $y = a + {}^{b}/_{a}x^2$ about the x –axis ($-a \le x \le a$). If the liquid enters at the end $x = -a$ of the pipe with velocity V, show that the time taken by a liquid particle to traverse the entire length of the pipe from $x = -a$ to $x = +a$ is

$$\frac{2a\left(1 + \frac{2}{3}b + \frac{1}{5}b^2\right)}{V(1+b)^2}$$

Assume that b is so small that the flow remains appreciably one dimensional throughout.

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27th AUGUST, 2008

MANAGEMENT - Paper-I

INSTRUCTIONS : Answers must be written in English only. Candidates should attempt at least 2 questions from each section and total 5 questions. The number of marks carried by each question is indicated at the end of the question.

Time: 3 hours — Marks: 300

SECTION – A

1. (a) Define Linear Programming. Discuss its application for decision making in industry and in National and Regional Planning. (20 Marks)

   (b) Solve the following LP-Problem through Simplex Method:

   Max. $Z = 20X_1 + 6X_2 + 8X_3$

   Subject to

   $$8X_1 + 2X_2 + 3X_3 \leq 200$$
   $$4X_1 + 3x_2 \leq 150$$
   $$2X_1 + X_3 \leq 50$$
   $$X_1, X_2, X_3 \geq 0$$

   (40 Marks)

2. (a) The information related to a project is given below:

Activity	Time	Activity	Time
1-2	4	5-6	4
1-3	1	5-7	8
2-4	1	6-8	1
3-4	1	7-8	2
3-5	6	8-10	5
4-9	5	9-10	7

   i) Draw the Project Network.
   ii) Determine the project completion time and identify the critical path.
   iii) Determine the Total Float of all activities.
   iv) Explain the importance of critical path and float in any project. (40 Marks)

   (b) Discuss the importance and usefulness of various types of forecasting techniques for managerial decisions at different hierarchical level. Give examples also. (20 Marks)

3. (a) Discuss the contribution and limitations of various approaches to management analysis. (30 Marks)

   (b) Identify and discuss the elements of external environment which affect the organisation immensely. Discuss the readymade garments export from India from that viewpoint. (30 Marks)

4. (a) What are the various types of decisions in any organisation? Discuss the information requirements for these decisions. Discuss briefly the decision making process under uncertainty. (30 Marks)

   (b) Define Vision, Mission and Objectives. Bring out the differences among them. Discuss all these in the context of Ministry of Railways. (30 Marks)

5. (a) A formal organisation is often conceived of as a communication system- Explain. Discuss the various types of communication channels that usually exist in any enterprise. (30 Marks)

(b) Define leadership. How are leadership theory and styles related to motivation? Discuss also briefly the relationship between leadership and managing change in the organisation. (30 Marks)

SECTION - B

6. (a) Discuss the concept of value chain with the help of diagram. Explain how it will help in improving competitiveness of the organisation. (30 Marks)

(b) Explain Activity Based Costing (ABC) with the help of an example. How is it different than existing traditional costing? (30 Marks)

7. (a) Critically analyse the policies of liberalisation, privatisation and globalisation as pursued by the Government of India since 1996. (30 Marks)

(b) Explain its impact on the management policy of public sector undertakings specifically of manufacturing sectors. (30 Marks)

8. (a) Explain the process of analysis of macro business environment with the help of an example from banking sector. (30 Marks)

(b) Discuss the changes in structural dimensions of Indian Economy during the last 20 years. (30 Marks)

9. (a) Define strategic Planning. Explain the Strategic Planning Process. (30 Marks)

(b) Explain the management control system for controlling global enterprise. (30 Marks)

10. Write short notes on any three of the following:

i) Zero based Budgeting
ii) Target Costing
iii) Management of Risk
iv) Analysis and Evaluation of Performance
v) SWOT Analysis.

(20+20+20 Marks)

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27th AUGUST, 2008

MANAGEMENT - Paper-II

INSTRUCTIONS : Answers must be written in English only. Candidates should attempt at least 2 questions from each section and total 5 questions. All questions carry equal marks.

Time: 3 hours Marks: 300

SECTION – A

Q.1. Write short notes on any three of the following (Each note not to exceed 200 words):

(i) Working Capital Management
(ii) Consumer Protection in India
(iii) WTO and its role in International Trade
(iv) Marketing Research
(v) Capital Markets in India

Q.2. What are the most commonly used tools of Marketing Communication? Which one of these tools is most effective for marketing of social themes and why? Elaborate your answer with the help of suitable examples.

Q.3. (a) What do you understand by the concepts of value and return? How are these concepts applicable to governmental projects in a country like India?

(b) Discuss the mechanism of regulation of Capital Markets as followed in India.

Q.4. (a) Discuss the role of IMF and World Bank in promoting international trade.

(b) How is international marketing strategy different from domestic marketing strategy? Discuss.

Q.5. Comment on the following statements (Each note not to exceed 200 words):

(i) "In governmental projects, Social Cost Benefit Analysis is more relevant as compared to hard financial return analysis"

(ii) "Advertising is a big business waste"

(iii) "In International Business, management of risk is not in our hands"

SECTION – B

Q.6. Write short notes on any three of the following (Each note not to exceed 200 words):

(i) Total Quality Management
(ii) Performance Measurement
(iii) Decision Support Systems
(iv) International Buying
(v) Industrial Relations in India

Q.7. Differentiate job analysis from job description. Discuss the relevance of these tools for effective Human Resource Management with the help of suitable examples.

Q.8. (a) What do you understand by the term "Capacity Planning"? Discuss this with the help of an example of a public system like a Railway Station .

(b) How are the principles of Supply Çhain Management applicable to a government system like Public Distribution System? Explain.

Q.9. (a) Discuss the role of Internet for developing economies with special emphasis on rural population.

(b) What are the various steps involved in System development Life Cycle? Discuss the outputs from each step.

Q.10. Comment on the following statements (Each note not to exceed 200 words):

(i) "Trade Unionism in India has undergone a significant change"
(ii) "IT can do for India what Oil has done for Kuwait"
(iii) "Codification is the key to Inventory Management"