

PARLIAMENT OF INDIA
(JOINT RECRUITMENT CELL)

MAIN EXAMINATION FOR POSTS OF EXECUTIVE/LEGISLATIVE/COMMITTEE/PROTOCOL OFFICER AND
RESEARCH/REFERENCE OFFICER IN LOK SABHA SECRETARIAT

3rd SEPTEMBER, 2010

MATHEMATICS - Paper-I

INSTRUCTIONS : This question paper consists of 10 questions in two sections A and B. Attempt at least two questions from each section and a total of five questions.
Questions must be answered in English only.

Time: 3 hours

Marks: 300

SECTION - A

Q1- (a) Show that $\lim_{x \rightarrow 0} \sin \frac{1}{x}$ does not exist. (15)

(b) Find all asymptotes of the curve:-

$$x^4 + a^4 = (y-a)^2 (x^2 - a^2) \quad (15)$$

(where $a > 0$ is a real constant).

(c) For what value of p does $\frac{\sin 2x + p \sin x}{x^3}$ tend to a finite limit as $x \rightarrow 0$? (15)

(d) Find the area enclosed by the curve $xy^2 = a^2(a-x)$ and y -axis. (15)

Q2- (a) If $u = f(r)$, where $r^2 = x^2 + y^2$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r). \quad (12)$$

(b) Differentiate $x^{\sin x}$ w.r.t $(\sin x)^x$. (12)

(c) If $u = \sec^{-1} \left(\frac{x^3 + y^3}{x + y} \right)$, show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \cot u \quad (12)$$

(d) If α and β are distinct real numbers, show that there exists a real number γ between α and β such that $\alpha^2 + \alpha\beta + \beta^2 = 3\gamma^2$. (12)

(e) Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\tan x}$

(12)

Q3 - (a) Evaluate

$$\iint_R e^{x^2+y^2} dy dx$$

where R is the semicircular region bounded by the x -axis and the curve $y = \sqrt{1-x^2}$. (15)

(b) Examine for convergence the improper integral

$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^2}$$

(15)

(c) If Γ denotes the gamma function, prove that

$$\sqrt{\pi} \Gamma(2m) = 2^{2m-1} \Gamma(m) \Gamma(m+\frac{1}{2})$$

(where $m > 0$).

(15)

(d) Find the greatest and smallest values that the function $f(x,y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$. (15)

Q4 - (a) Find the value of λ so that the equation

$$x^2 - \lambda xy + 2y^2 + 3x - 5y + 2 = 0$$

may represent a pair of straight lines and find the lines. (15)

(b) Obtain the equation of the sphere which passes through the three points $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ and has its radius as small as possible. (15)

(c) Find the equation of the cone which passes through the common generators of the cones $-2x^2 + 4y^2 + z^2 = 0$ and $10xy - 2yz + 5zx = 0$ and the line with direction cosines proportional to $1, 2, 3$. (15)

(d) Find the equation of the right circular cylinder whose axis is $x = 2y = -z$ and radius is 4. (15)

Q5- Attempt any five parts : (12x5 = 60).

(a) Show that the vectors (a_1, a_2) and (b_1, b_2) in \mathbb{C}^2 are linearly dependent if and only if $a_1 b_2 = a_2 b_1$, \mathbb{C} denoting the space of all complex numbers. (12)

(b) Extend the set $S = \{(1, 1, 0)\}$ to form two different basis of $\mathbb{R}^3(\mathbb{R})$. (12)

(c) Let $A = \{(x, y, 0) : x, y \in \mathbb{R}\}$ and $B = \{(0, y, z) : y, z \in \mathbb{R}\}$ be two subspaces

of $V = \mathbb{R}^3(\mathbb{R})$. Find the dimension of $A+B$.

(12)

(d) Define the rank of a matrix. Find the rank of the matrix

$$\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

(12)

(e) If A and B are Hermitian matrices such that $A^2 + B^2 = 0$, show that $A=0$ and $B=0$.

(12)

(f) Describe explicitly a linear transformation from \mathbb{R}^3 to \mathbb{R}^3 which has its range the subspace spanned by the vectors $(1, 0, -1)$ and $(1, 2, 2)$.

(12)

SECTION - B

Q6-(a) Find the differential equation corresponding to

$$y = ae^{2x} + be^{-3x} + ce^x$$

where a, b, c are arbitrary constants. (15)

(b) Solve:

$$x \frac{dy}{dx} + y = y^2 x^3 \cos x. \quad (15)$$

(c) Reduce the differential equation

$$(x^2 y - 2xy^2) dx - (x^3 - 3x^2 y) dy = 0$$

into an exact differential equation and hence solve it. (15)

(d) If p denotes $\frac{dy}{dx}$, solve

$$y = 2px + f(xp^2). \quad (15)$$

Q7-(a) Use the method of variation of parameters

to solve:

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} = e^x \sin x. \quad (15)$$

(b) Solve:

$$x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right) \quad (15)$$

(c) Solve:

$$x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 3y = 2x^5. \quad (15)$$

(d) Solve:

$$e^{3x} (p-1) + p^3 e^{2y} = 0, \quad \text{where } p \equiv \frac{dy}{dx}. \quad (15)$$

Q8-(a) A ladder leans against a smooth wall, the lower end resting on a rough floor for which the coefficient of friction is $\frac{1}{4}$. Find the inclination of the ladder to the vertical if it is just on the point of slipping. (15)

(b) Consider a system with two degrees of freedom for which the potential energy is given by

$$V = q_1^2 + 3q_1 q_2 + 4q_2^2.$$

where q_1 and q_2 are the generalized coordinates of the system. Determine the position of equilibrium and discuss its stability. (15)

(c) Show that, if a rigid body is in equilibrium under the action of four forces, the invariant $(\vec{F} \cdot \vec{G})$ of any two

is equal to the invariant $(\vec{F} \cdot \vec{G})$ of the other two. Show also that the invariant $(\vec{F} \cdot \vec{G})$ of any three of the forces is zero. (15)

(d) Find the thrust on a vertical quadrilateral which has one side of length a in the surface, and the opposite side of length b parallel to it at a depth h . Also, if the fluid consists of top layer of density ρ and thickness $\frac{h}{2}$, and the rest of density σ , prove that the thrust on the quadrilateral is

$$\frac{gh^2}{48} \left\{ (7a+11b)\rho + (a+5b)\sigma \right\}. \quad (15)$$

Q9- (a) Two weights w_1 and w_2 rest on a rough plane inclined at an angle α to the horizontal and are connected by a string which lies along the line of greatest slope. If μ_1 and μ_2 are their coefficients of friction with the plane, and $\mu_1 > \tan \alpha > \mu_2$, prove that, if they are both on the point of slipping,

$$\tan \alpha = \frac{\mu_1 w_1 + \mu_2 w_2}{w_1 + w_2} \quad (15)$$

(b) A plane triangular area is immersed in liquid of uniform density with its plane vertical, one side horizontal and the opposite corner downwards. Its vertical altitude is h , and the horizontal side is at a depth h below the effective surface. Show that its centre of pressure is at a depth $\frac{11}{8}h$ below the surface. (15)

(c) Prove that a necessary and sufficient condition for a field \vec{F} to be conservative is that force is gradient of potential energy with its sign reversed. (15)

(d) A cable, of weight w per unit length and length $2l$ hangs from two points A and B, at the same height and at a distance $2a$ apart. Show that the maximum tension in the cable is $wa \sqrt{\frac{a}{6(l-a)}}$. (15)

Q10- (a) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$ where t is the time. Find the components of its velocity and acceleration at $t=1$ in the direction

$$\hat{i} - 3\hat{j} + 2\hat{k}.$$

(12)

(b) If \vec{A} and \vec{B} are irrotational, prove that $\vec{A} \times \vec{B}$ is solenoidal. (12)

(c) Verify Green's theorem in the plane for

$$\int_C (x^2 - xy^3) dx + (y^2 - 2xy) dy$$

where C is the square with vertices $(0,0)$, $(2,0)$, $(2,2)$, $(0,2)$. (12)

(d) State Stoke's theorem. Verify Stoke's theorem for $\vec{A} = (2x-y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, where S is the upper surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. (12)

(e) Calculate the curvature and torsion of the curve $\vec{r} = \vec{r}(u)$ in terms of u , where $\vec{r}(u) = (u, u^2, u^3)$. (12)

PARLIAMENT OF INDIA
(JOINT RECRUITMENT CELL)

MAIN EXAMINATION FOR POSTS OF EXECUTIVE/ LEGISLATIVE/
COMMITTEE/PROTOCOL OFFICER AND RESEARCH/REFERENCE OFFICER IN
LOK SABHA SECRETARIAT

3rd SEPTEMBER, 2010

MATHEMATICS-PAPER-II

INSTRUCTIONS: Answers must be written in English only. Candidates should attempt at least 2 questions from each section and total 5 questions. All questions carry equal marks. All the symbols having their usual meaning.

Time: 3 hours

Marks: 300

SECTION A

1. (a) Let P be a Sylow p -subgroup of G . Then any conjugate gpg^{-1} is also a Sylow p -subgroup of G .
(b) State and prove Cayley's theorem for permutation groups.
(c) Let R be a commutative ring and A an ideal of R . Then $\sqrt{A} = \{a \in R \mid a^n \in A, \text{ for some } n \geq 1\}$ is an ideal of R .

2. (a) State and prove implicit function theorem.
(b) Test the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n}$ for uniform convergence on $[0,1]$.
(c) Let $F=2xzi-xj+y^2k$, evaluate $\iiint_V F \cdot dv$ where V is the region bounded by the surface $x=0, y=0, y=6, z=x^2, z=4$.

3. (a) Evaluate $\oint_c \frac{e^z dz}{z(1-z)^3}$ where c is (i) $|z| = \frac{1}{2}$, (ii) $|z-1| = \frac{1}{2}$, (iii) $|z| = 2$.
(b) State and prove Residue theorem. Using this theorem, Evaluate $\oint_c \frac{z^2+4}{z^3+2z^2+z} dz$ where c is (i) $|z|=1$, (ii) $|z+1|=1$, (iii) $|z-1|=5$.

4. (a) Use duality to solve the following LPP:
Maximize $Z=2x_1+x_2$
Subject to the constraints:
 $x_1+2x_2 \leq 10, x_1+x_2 \leq 6$
 $x_1-x_2 \leq 2, x_1-2x_2 \leq 1$
 $x_1, x_2 \geq 0$

- (b) Solve the following transportation problem.
Is the optimal solution obtained by you is unique one? If not, why? What are the alternate optima then?

Source	Destination				Supply
	1	2	3	4	
1	15	18	22	16	30
2	15	19	20	14	40
3	13	16	23	17	30
Demand	20	20	25	35	100

5. (a) The sub group H of a group G is normal iff $g^{-1}Hg = H \forall g \in G$.
 (b) Prove that $f(x) = \sin x$ is Riemann integrable on $[0, \pi/2]$.
 (c) Examine the convergence and absolute convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n^2 + 1}$

SECTION-B

6. (a) Find the complete integral of $p^2x + q^2y = z$
 (b) Consider the problem of heat conduction in a finite rod of length 'l' which is governed by the following initial and boundary value problem:
 $u_t = u_{xx}, 0 < x < l, t > 0$
 $u(0, t) = u(l, t) = 0, t > 0$
 $u(x, 0) = x(l - x), 0 \leq x \leq l$

Find the solution of this problem by the method of separation of variables.

7. (a) Use Gauss-Seidel method to solve the equations correct up to 4 decimal places by performing 7 iterations

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$

$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$

$$-x_1 - x_2 + 10x_3 - 2x_4 = 27$$

$$-x_1 - x_2 - 2x_3 + 10x_4 = -9$$

- (b) Find $y(0.2)$, $y(0.4)$, $y(0.6)$ by using the Runge-Kutta fourth order method given that

$$\frac{dy}{dx} = 1 + y^2, y(0) = 0.$$

8. (a) Define holonomic and Non-holonomic systems. Give two examples of each of the system and explain.
 (b) Derive the Hamiltonians equations of motion. Discuss the motion of a spherical pendulum using the Hamiltonian equations.
 (c) Derive the Lagranges equation of second kind for a holonomic system in independent co-ordinates. Discuss the motion of Harmonic oscillator by using the Lagranges equation of second kind.
9. (a) Define a two-dimensional motion. what are its characteristics? What do you mean by a point as well a curve in a two-dimensional motion?
 (b) Define a source and its strength of a two-dimensional motion. find the relation between the strength of a source and radial component of the velocity.
 (c) Find the complex potential of a source of strength 'm' situated at the origin. Find the streamlines, potential lines and circulation of a two dimensional source flow.
10. (a) Consider the Navier- Stokes equation in the following form:

$$\rho \frac{D\bar{q}}{Dt} = \rho \bar{F} - \text{grad}p + \mu \nabla^2 \bar{q} + \frac{\mu}{3} \text{grad}(\text{div} \bar{q}).$$

Express this equation in x,y, and z directions. How does this equation reduces for an incompressible flow? Consequently deduce Euler's equation of motion by assuming suitable assumptions.

- (b) Explain Octal and Hexadecimal systems. Give flow chart for Newton - Raphson method.

- (c) Verify $yzdx + (x^2y - zx)dy + (x^2z - xy)dz = 0$ is integrable and find its integral.